Leptgenesis from Bilinear R-parity Violating Couplings

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Abstract

We reexamine the idea that bilinear R-parity violating couplings can be responsible for leptogenesis. We prove that, to have lepton number violation before the electroweak phase transition, misalignment between the lepton-Higgsino and slepton-Higgs sectors must be involved. The processes that generate a lepton asymmetry are bino or wino decays into a lepton-Higgs and slepton-Higgsino final states. Since these decays occur mainly while the gauginos are still in thermal equilibrium, they quantitatively fail to produce the observed baryon asymmetry of the universe.

1 Introduction

The observed baryon asymmetry is [1]:

$$B \equiv \frac{n_b - n_{\bar{b}}}{s} = \frac{n_b}{s} \simeq 10^{-10}. \tag{1}$$

It is possible that the origin of this asymmetry is a lepton asymmetry, which is partially converted into a baryon asymmetry by sphaleron processes [2]. This scenario is known as leptogenesis. Within the framework of the supersymmetric standard model (SSM), the required lepton number violation can be induced by bilinear R-parity violating couplings [3, 4, 5]. We critically examine this idea, paying special attention to the condition that the lepton number violating decays must occur out of equilibrium [6].

Within the SSM, the final baryon asymmetry is related to the initial lepton asymmetry by $B = -\frac{32}{92}L$ [7, 8]. The lepton asymmetry can be expressed as a product of three factors: the ratio

between the number density of the decaying particle n_i and the entropy density $g_* n_\gamma$ (g_* denotes the effective number of relativistic degrees of freedom), the branching ratio into lepton number violating modes $\mathcal{B}_i^{\not{L}}$, and the CP asymmetry in these decays ϵ_i:

$$L \simeq \mathcal{B}_i^{\not{L}} \times \epsilon_i \times \frac{n_i}{g_* n_\gamma}. \tag{2}$$

We will estimate each of these three factors in order to obtain an estimate or, more precisely, an upper bound, on the lepton asymmetry so induced. The plan of this paper is as follows. In Section 2 we define the model, and find the sources of CP and L violation. In Section 3 we estimate the branching ratio $\mathcal{B}_i^{\not{L}}$. In Section 4 we find the CP asymmetry ϵ_i. In Section 5 we obtain an upper bound on n_i at the time of departure from thermal equilibrium. In section 6 we obtain an upper bound on the lepton asymmetry generated in this model and conclude.

The idea of leptogenesis from bilinear R-parity violating couplings was originally proposed and investigated in refs. [3, 4, 5]. We disagree with their results in several points and, in particular, the final answer on whether this could be a successful scenario for baryogenesis.

2 CP and Lepton Number Violation

We work in the framework of the SSM without R-parity [$R_p = (-1)^{3(B-L)+2s}$]. We follow the notation and methods of refs [9, 10]. The SSM has four chiral supermultiplets that are doublets of $SU(2)$ and carry hypercharge $-1/2$. These are the "down Higgs" H_d and the "lepton-doublet" L_i superfields. In the absence of R-parity, there is no quantum number to distinguish the Higgs from the leptons, so we denote these fields collectively as L_α, with $\alpha = 0, 1, 2, 3$. We neglect trilinear R-parity violating terms in the superpotential and in the soft SUSY breaking part of the Lagrangian. The relevant part of our lagrangian is the following:

$$\begin{aligned}
\mathcal{L} \quad &\supset \quad \left(\imath\frac{g_1}{\sqrt{2}}\tilde{B} + \imath\sqrt{2}g_2\tilde{W}^a T^a\right)\left(H_u^\dagger \tilde{H}_u + L_\alpha^\dagger \tilde{L}_\alpha\right) - \frac{1}{2}\mu_\alpha \tilde{L}_\alpha \tilde{H}_u \\
&- \quad \left(|\mu_\alpha|^2 + m_{H_u}^2\right)|H_u|^2 - \left(m_{\alpha\beta}^2 + \mu_\alpha \mu_\beta^\dagger\right) L_\alpha L_\beta^\dagger - b_\alpha L_\alpha H_u \\
&- \quad \frac{1}{8}(g_1^2 + g_2^2)\left(|H_u|^2 - |L_\alpha|^2\right)^2 - \frac{1}{2}m_{\tilde{B}}\tilde{B}\tilde{B} - \frac{1}{2}m_{\tilde{W}}\tilde{W}^a\tilde{W}^a + h.c.
\end{aligned} \tag{3}$$

where $\tilde{X}(X)$ denotes fermions (scalars). Without loss of generality, we can choose a basis in which $\mu_\alpha = \mu(1, 0, 0, 0)$ and $m_{I\neq J}^2 = 0$ where $I, J = 1, 2, 3$. With additional phase rotations to remove unphysical phases, we finally have the following lagrangian:

$$\begin{aligned}
\mathcal{L} \quad &\supset \quad [\imath\frac{g_1}{\sqrt{2}}L_\alpha^\dagger \tilde{L}_\alpha \tilde{B} + \imath\frac{g_1}{\sqrt{2}}H_u^\dagger \tilde{H}_u \tilde{B} + \imath\sqrt{2}g_2 e^{i\phi_g} L_\alpha^\dagger \tilde{L}_\alpha T^a \tilde{W}^a + \imath\sqrt{2}g_2 e^{i\phi_g} H_u^\dagger \tilde{H}_u T^a \tilde{W}^a \\
&- \quad |b_\alpha| L_\alpha H_u - \frac{1}{2}|\mu| e^{i\phi_0}\tilde{L}_0 \tilde{H}_u - |m_{0I}^2| e^{i\phi_I} L_0^\dagger L_I \\
&- \quad \frac{1}{2}|m_{\tilde{B}}|\tilde{B}\tilde{B} - \frac{1}{2}|m_{\tilde{W}}|\tilde{W}^a\tilde{W}^a + h.c] + \text{terms with real couplings.}
\end{aligned} \tag{4}$$

The five physical phases are $\phi_g = \frac{1}{2} arg\left(\frac{m_{\tilde{B}}}{m_{\tilde{W}}}\right), \phi_0 = arg\left(\frac{\mu m_{\tilde{B}}}{b_0}\right)$ and $\phi_I = arg\left(\frac{m_{0I}^2 b_0}{b_I}\right)$. The CP violating phase that would turn out to induce leptogenesis is ϕ_g.

We now identify the conditions for lepton number violation. Since R-parity is violated here only by bilinear terms, lepton number violation will be induced by the mass matrices. Note that at temperatures above the electroweak phase transition (EWPT), the SU(2) symmetry is unbroken ($\langle H_u \rangle = \langle L_\alpha \rangle = 0$). Different SU(2) multiplets do not mix. Thus the question of whether lepton number is violated can be examined by considering the SU(2) doublets alone. The fermion mass matrix in the basis $(\tilde{H}_u, \tilde{L}_\alpha)$ has the form

$$M^f = \begin{pmatrix} 0 & \mu & 0_{1\times3} \\ \mu & 0 & 0_{1\times3} \\ 0_{3\times1} & 0_{3\times1} & 0_{3\times3} \end{pmatrix}$$

Thus, defining $\tilde{L}_0$ (the field that has a μ-term that couples it to H_u) as the Higgs field $\tilde{H}_d$, and the three $\tilde{L}_I$ fields (the ones orthogonal to the direction of μ) as leptons, we learn that lepton number is conserved in the fermion sector. In other words, the leptons are massless above the EWPT while the Higgsinos are massive.

Consider now the mass-squared matrix of the (H_u, L_α) scalars:

$$M^s = \begin{pmatrix} |\mu|^2 + m_{H_u}^2 & b_0 & b_I \\ b_0 & |\mu|^2 + m_{H_d}^2 & m_{I0}^2 \\ b_I^\dagger & m_{I0}^{2\dagger} & m_{L_I}^2 \end{pmatrix}$$

The lepton number, defined above, is violated by the b_I and m_{I0}^2 terms. We conclude that, for lepton number to be violated before the EWPT, one must consider both the scalar and fermionic sectors of the model. The basis invariant statement is that, in the SU(2)-doublet sector, a misalignment between the fermion and the scalar mass matrices is the source of lepton number violation. In [3, 4, 5] the authors neglected the mixing in the bosonic sector which leads to a conserved lepton number.

3 The Branching Ratio $\mathcal{B}^{\not{L}}$

To calculate the branching ratio of lepton number violating gaugino decays, we switch to the mass basis:

$$\begin{pmatrix} H'_u \\ H'_d \\ L'_I \end{pmatrix} = U^s \times \begin{pmatrix} H_u \\ H_d \\ L_I \end{pmatrix}, \qquad \begin{pmatrix} \tilde{H}'_u \\ \tilde{H}'_d \\ \tilde{L}'_I \end{pmatrix} = U^f \times \begin{pmatrix} \tilde{H}_u \\ \tilde{H}_d \\ \tilde{L}_I \end{pmatrix}. \tag{5}$$

Here U^f and U^s are the unitary matrices which diagonalize M^f and M^s, respectively, and primes denote mass eigenstates. In this basis, the gaugino-matter couplings can be written as follows:

$$\imath\sqrt{2} g_{(k)} T^a \tilde{\lambda}^a_{(k)} A_l^\dagger \delta_{lk} \psi_k = \imath\sqrt{2} g_{(k)} T^a \tilde{\lambda}^a_{(k)} A'^\dagger_m V_{mn} \psi'_n \tag{6}$$

where $\tilde{\lambda}^a$ denote gauginos, A' and ψ' denote, respectively, scalar and fermion matter fields. Before the EWPT, the gauginos do not mix, so that $\tilde{\lambda}^a$ stands for both the interaction and mass eigenstates. The mixing matrix $V = U^s U^{f\dagger}$ is unitary. The important point is that if the M^s and M^f matrices were aligned, namely b_I and m_{0I}^2 had vanished in the basis where $\mu_I = 0$, then U^s and U^f would

be both block diagonal, with $(U^s)_{LH} = (U^f)_{LH} = 0$ (by L we mean here any of the three L_I and by H we refer to H_u and H_d), and so would V. With misaligned matrices, we have lepton number violating elements, $V_{LH} \neq 0$. These lead to gaugino decays which violate lepton number:

$$\mathcal{B}^{\not L} \equiv BR(\tilde{\lambda} \to LH) \simeq \frac{\sum_{L,H} |V_{LH}|^2}{5} \tag{7}$$

where $\tilde{\lambda} = \tilde{B}$ or $\tilde{W}$ and, if the masses of the decay products are neglected, $\mathcal{B}^{\not L}_{\tilde{B}} = \mathcal{B}^{\not L}_{\tilde{W}}$.

The lepton number violating matrix elements, V_{LH}, are of $\mathcal{O}(b_I/\tilde{m}^2)$ and $\mathcal{O}(m^2_{I0}/\tilde{m}^2)$, where $\tilde{m}$ is the scale of the soft supersymmetry breaking terms. From phenomenological constraints, such as neutrino masses [11, 12], we know that various lepton number violating couplings must be suppressed by, at least, a factor of $\mathcal{O}[(m_\nu/m_Z)^{1/2}]$ [10]. Making the natural assumption that R-parity is an approximate symmetry broken by small parameters $\leq \mathcal{O}(10^{-4})$, we conclude that, very likely, $|V_{LH}| \leq \mathcal{O}(10^{-4})$ and, consequently,

$$\mathcal{B}^{\not L} \leq \mathcal{O}(10^{-8}). \tag{8}$$

With fine-tuning, however, this constraint can be avoided.

4 The CP Asymmetry ϵ

The CP asymmetry generated from gaugino decays is defined as follows:

$$\epsilon_i \equiv \frac{\Gamma(\tilde{\lambda}_i \to \tilde{L}H^\dagger) + \Gamma(\tilde{\lambda}_i \to \tilde{H}L^\dagger) - \Gamma(\bar{\tilde{\lambda}}_i \to \overline{\tilde{L}H^\dagger}) - \Gamma(\bar{\tilde{\lambda}}_i \to \overline{\tilde{H}L^\dagger})}{\Gamma(\tilde{\lambda}_i \to \tilde{L}H^\dagger) + \Gamma(\tilde{\lambda}_i \to \tilde{H}L^\dagger) + \Gamma(\bar{\tilde{\lambda}}_i \to \overline{\tilde{L}H^\dagger}) + \Gamma(\bar{\tilde{\lambda}}_i \to \overline{\tilde{H}L^\dagger})} \tag{9}$$

This asymmetry is calculated by evaluating the imaginary part of the interference term between tree-level and one-loop diagrams. Neglecting the masses of the decay products, we obtain an upper bound on ϵ which serves also as a good estimate if there is no strong phase space suppression.

For the CP asymmetry generated in $\tilde{B}$ decays, we obtain

$$\epsilon_{\tilde{B}} \lesssim 6\alpha_2 \sin(2\phi_g)\sqrt{y_W}\left[\frac{5}{2(1-y_W)} + 1 - (1+y_W)\ln\left(\frac{1+y_W}{y_W}\right)\right], \tag{10}$$

where $y_W = m^2_{\tilde{W}}/m^2_{\tilde{B}}$. For $y_W \in [0.5, 1.5]$ we get $\epsilon_{\tilde{B}} \lesssim 1$ and $\epsilon_{\tilde{B}}$ will be estimated as such. The CP asymmetry generated in $\tilde{W}$ decays, $\epsilon_{\tilde{W}}$, can be obtained straightforwardly from the expression for $\epsilon_{\tilde{B}}$ by replacing $\alpha_2 \to \alpha_1$ and $y_W \to y_W^{-1}$.

Finally, we note that the process $\tilde{W}'_3 \leftrightarrow L^\pm W^\mp$ [3, 4, 5] does not occur at tree-level because at high temperatures, before the EWPT, the gauginos do not mix with the other neutralinos. Related processes, such as $\tilde{W}'_3 \leftrightarrow L^\pm W^\mp H'_{u,d}$, can occur at higher order.

5 Departure from Thermal Equilibrium: $n_{\tilde{B},\tilde{W}}/n_\gamma$

The out-of-equilibrium condition reads

$$\Gamma_{\tilde{X}} < H(T = m_{\tilde{X}}) = 1.66 g_*^{1/2} (m_{\tilde{X}}^2 / M_{pl}), \tag{11}$$

where H is the Hubble constant, and $\Gamma_{\tilde{X}}$ is the decay rate of the gaugino. For the bino, the decay rate is given by

$$\Gamma_{\tilde{B}} \sim \frac{5\alpha_1 m_{\tilde{B}}}{8} \begin{cases} m_{\tilde{B}}/T & T \gtrsim m_{\tilde{B}} \\ 1 & T \lesssim m_{\tilde{B}} \end{cases} \tag{12}$$

while for the wino, one has to substitute $\alpha_1 \to \alpha_2$, $m_{\tilde{B}} \to m_{\tilde{W}}$ and $8 \to 2$. Putting these expressions into eq. (11), we find that the out of equilibrium condition is far from being satisfied for gaugino masses of order a few TeV or less. Hence the bino and wino decouple much below the temperature $T \sim m_{\tilde{X}}$ that is required in order to have sufficient overabundance. We conclude that, at the time of departure from thermal equilibrium, $n_{\tilde{B},\tilde{W}}/n_\gamma$ is exponentially suppressed and is well below $\mathcal{O}(1)$. We now make a more quantitative estimate of this ratio.

Let us examine the processes that wash-out the lepton asymmetry that is generated in gaugino decays. These processes are inverse decays, annihilations and $2 \leftrightarrow 2$ scattering processes mediated by intermediate gauginos. We follow similar considerations made in refs. [13, 14]. It turns out that the most significant process is the inverse decays. (This result is in accordance with the Boltzman equations analysis carried out in refs. [4, 5].) Thus we estimate the freeze-out temperature T_f as the one where inverse decays (ID) become ineffective, $\Gamma_{ID}(T_f) = H(T_f)$. We obtain

$$T_f \lesssim m_{\tilde{X}}/36. \tag{13}$$

The ratio $T_f/m_{\tilde{X}}$ is rather insensitive to variation of $m_{\tilde{X}}$ in the range of $10^2 - 10^5$ GeV.

We make the approximation that from this moment and on the gauginos propagate freely in the universe until they decay. Since the temperature at the time of decay, $T_d^{\tilde{X}}$ defined by $H(T_d^{\tilde{X}}) \sim \Gamma_{\tilde{X}}$, fulfills $T_d^{\tilde{X}} \gg m_{\tilde{X}} > T_f$, then the decays will be extremely rapid. We can thus safely assume that all remaining gauginos decay instantly at T_f. This approximation neglects the small fraction of the asymmetry generated prior to T_f, and the small fraction of the asymmetry that is washed out by inverse decays and scattering that rarely occur at $T \lesssim T_f$. The number density at the time of freeze-out is then estimated as follows:

$$\left.\frac{n_{\tilde{X}}}{n_\gamma}\right|_{T=T_f} = \frac{2}{\frac{2\zeta(3)}{\pi^2} T_f^3} \left(\frac{m_{\tilde{X}} T_f}{2\pi}\right)^{\frac{3}{2}} e^{-m_{\tilde{X}}/T_f}. \tag{14}$$

Given the smallness of $T_f/m_{\tilde{X}}$, we find a very strong suppression factor:

$$\left.\frac{n_{\tilde{X}}}{n_\gamma}\right|_{T=T_f} \sim 10^{-14}. \tag{15}$$

6 Conclusions

We have analyzed the lepton asymmetry induced by bino and wino decays with bilinear R-parity violating couplings. We obtained the following results:

1. Lepton number violation at temperatures above the EWPT requires misalignment between the lepton-Higgsino and slepton-Higgs mass matrices. The resulting branching ratio is expected to be small,

$$\mathcal{B}^{\not L}_{\tilde{B},\tilde{W}} \lesssim 10^{-8}, \tag{16}$$

unless fine-tuning is involved.

2. The CP asymmetry is suppressed by a loop factor (α_1 or α_2), but this could be compensated by other factors of order one:

$$\epsilon_{\tilde{B},\tilde{W}} \lesssim 1. \tag{17}$$

3. The gauge interactions cause the gauginos to depart from thermal equilibrium at very late time, when their number density is strongly suppressed:

$$n_{\tilde{B},\tilde{W}}/s \sim 10^{-16}. \tag{18}$$

This strong suppression factor related to the late departure from thermal equilibrium is effective also in models with trilinear R-parity violating couplings. Actually, this situation has a much more general validity: If a decaying particle is not a singlet of the Standard Model gauge group, it will depart from thermal equilibrium at a temperature well below its mass (unless $M \gtrsim 10^{14}\ GeV$). Its number density at the time of freeze-out would be too small to produce the observed baryon asymmetry (see *e.g.* [15]).

We can thus estimate that the baryon number so generated is very small,

$$B \lesssim 10^{-25}(\mathcal{B}^{\not L}/10^{-8}), \tag{19}$$

to be compared with the observed value, $B \sim 10^{-10}$. These results prove that this model cannot account for the observed baryon asymmetry of the universe. From a cosmological point of view this failure adds up to the fact that R-parity violating interactions mean that the LSP cannot serve as a good dark matter candidate. Hence, from a cosmologist point of view, R-parity violating models are more problematic (and less predictive) than R-parity conserving ones.

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